15.1a Diagmagnetism arises from the fundamental quantum mechanical response of electron orbitals to external magnetic fields. When an external field is applied, Lenz's law manifests at the atomic scale, generating induced currents that oppose the applied field.

The diagmagnetic susceptibility can be derived from quantum perturbation theory, where the external magnetic field acts as a perturbation to the electron's Hamiltonian. Starting with the quantum mechanical expression:-

$$X_m = \frac{M}{H} = \frac{\mu_0 q^2 Z r^2}{4 m_e V}$$

Where:

- q represents the elemantary charge
- Z is the number of electrons per atom
- r^2 is the mean square radius of the electron orbit.
- m_e is the electron mass
- V is the atomic volume

For a typical solid, we can estimate these parameters:

- $q = 1.602 \times 10^{-19}$ C
- $Z \simeq 1$ (considering primarily outer shell electrons)
- $r \simeq 10^{-10}$ m (approximate atomic radius)
- $m_e = 9.11 \times 10^{-31}$ kg
- $V \doteq 10^{-30}$ m^3 (typical atomic volume)

Substituting these values:

$$X_m = -\mu_0 \frac{(1.602 \times 10^{-19})^2 \cdot (1) \cdot (10^{-10})^2}{4 \cdot (9.11 \times 10^{-31}) \cdot (10^{-30})}$$

$$X_m = -(4\pi \times 10^{-7}) \frac{(1.602 \times 10^{-19})^2 \cdot (10^{-20})}{4 \cdot (9.11 \times 10^{-31}) \cdot (10^{-30})}$$

$$\chi_m \approx -10^{-4}$$

This negative susceptibility is characteristic of diamagnetic materials, indicating that they generate an internal magnetization that opposes the applied field. The small magnitude explains why diagmagnetism is a relatively weak effort compared to paramagnetism or ferromagnetism, and is often overshadowed by these stronger magnetic behaviours when they are present.

15.1 b

The force experienced by a diamagnetic material in a non-uniform magnetic field can be derived from the principle of virtual work. The magnetic potential energy of a material with volume V in a magnetic field H is:

$$U = -\int \vec{M} \cdot \vec{B} \, dV$$

For a diamagnetic material, $\vec{M} = \chi_m \vec{H}$, and the force is the negative gradient of this energy:

$$\vec{F} = -\nabla U = -\nabla \left(-\int \chi_m \vec{H} \cdot \vec{B} \, dV \right)$$

For a uniform susceptibility and assuming $\vec{B} = \mu_0 \vec{H}$:

$$\vec{F} = -V \chi_m \nabla \left(\frac{1}{2} \mu_0 H^2 \right)$$

For a field gradient in the z-direction:

$$F_z = -V \chi_m \mu_0 H \frac{dH}{dz}$$

For levitation of a frog with mass m, this force must counteract gravity:

$$F_z = mg$$

Therefore:

$$-V \chi_m \mu_0 H \frac{dH}{dz} = mg$$

Rearranging to solve for the required field gradient:

$$H \frac{dH}{dz} = \frac{-mg}{V \chi_m \mu_0}$$

For a small frog:

- Mass $m \approx 0.1$ kg
- Volume $V \approx 0.1 \, L = 10^{-4} \, m^3$
- Accelerating due to gravity $g = 9.8 \, m/s^2$
- Diagmagnetic susceptibility $\chi_m \approx -10^{-4}$ (from part a)

Substituting :

$$H\frac{dH}{dz} = \frac{-(0.1kg)(9.8m/s^2)}{(10^{-4}m^3)(-10^{-4})(4\pi \times 10^{-7} H/m)}$$

$$H\frac{dH}{dz} = \frac{0.98}{4\pi \times 10^{-15}} \approx 7.8 \times 10^{13} \ A^2/m^3$$

If we assume the field gradient occurs over a distance comparable to the frog's size ($\approx 0.1m$), then :

$$H^2 \approx 7.8 \times 10^{13} \ A^2/m^2 \times 0.1m = 7.8 \times 10^{12} \ A^2/m^2$$

$$H \approx 8.8 \times 10^6 \ A/m$$

Converting the magnetic flux density :

$$B = \mu_0 H = (4\pi \times 10^{-7})(8.8 \times 10^6) \approx 11 \ T$$

This field strength is achievable with modern superconducting magnets, explaining why magnetic levitation of diamagnetic objects (including frogs) has been successfully demonstrated in laboratory settings.

15.2 a The energy stored in a magnetic field can be expressed as:

$$E = \frac{1}{2\mu_0} \int B^2 \, dV$$

This equation represents the energy density integrated over the volume containing the field. For magnetic materials, this energy depends on the material's permeability and the applied field strength.

When a permanent magnetic interacts with an unmagnetized ferromagnet, the system seeks to minimize its total energy. The field from the permanent magnet tries to minimize its energy by increasing the effective permeability of its surroundings, which produces an attractive force between the permanent magnet and the ferromagnet.

The ferromagnetic material, having a much higher permeability ($\mu \gg \mu_0$), provides a low-reluctance path for the magnetic flux. This concentrates the field lines within the ferromagnet reducing the overall magnetic field energy in the system. The gradient of this energy reduction manifests as the attractive force between the two objects.

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15.3b

The attraction between opposite poles of permanent magnets can be understood through the lens of energy minimization in dipole-dipole interactions. The energy of a magnetic dipole $\vec{m}$ in a magnetic field $\vec{B}$ is:

$$U = -\vec{m} \cdot \vec{B}$$

For two dipoles, the field from one dipole interacts with the other, creating a configuration-dependant energy landscape. The system naturally evolves towards the lowest energy state.

When opposite poles face each other, the dipoles align antiparallel to each other but parallel to the line connecting them. This configuration minimizes the energy of interaction, resulting in an attractive force.

Mathematically, the energy is minimized when the return flux from one magnet points in the direction of the remnant magnetization of the other. This creates a coherent flux path through both magnets, reducing the overall magnetic field energy in the surrounding space.

The fundamental principle at work is that magnetic system evolve toward configurations that minimize the total field energy, which often means creating closed magnetic flux paths with minimal external field.

15.5a

Magnetic hysteresis represents the path-dependant relationship between the applied magnetic field H and the resulting magnetization M (or flux density B) in ferromagnetic materials. This non-linear relationship is fundamental to understanding energy storage and dissipation in magnetic systems.

The equation $M \rightarrow \frac{1}{\mu_0}(B-H)$ describes the relationship between magnetization, flux density, and field strength. In ferromagnetic materials, this relationship follows different paths depending on whether the field is increasing or decreasing, creating a hysteresis loop.

The area enclosed by the hysteresis loop represents energy dissipated as heat during a complete magnetization cycle. This energy loss is crucial in applications like transformers and motors, where it contributes to inefficiency.

For energy dissipated around a loop:

$$\oint \frac{\partial U}{\partial t}\, dt = \oint H \cdot \frac{\partial B}{\partial t}\, dt = \oint H \cdot dB$$

This integral equals the area enclosed by the B-H curve, representing energy dissipated per unit volume per cycle.

15.5b Calculating power Dissipation

For a square hysteresis loop with ~~coercitivity~~ coercivity H_c and saturation flux density B_s, the energy dissipated per cycle is:

$$E_{dissipated} = \oint H \cdot dB \approx 4 H_c B_s$$

Given:

- $H_c = 4 \times 10^3$ A/m
- $B_s = 1.57$ T

The energy dissipated per cycle per unit volume is:

$$E_{dissipated} = 4 \times (4 \times 10^3 \text{ A/m}) \times (1.57 \text{ T}) = 2.51 \times 10^4 \text{ J/m}^3$$

For a transformer core operating at 60 Hz, the power dicsipation per unit volume would be: $P = E_{dissipated} \times f = 2.51 \times 10^4 \text{ J/m}^3 \times 60 \text{ Hz} = 1.51 \times 10^6 \text{ W/m}^3$

This significant power loss explains why transformer cores are typically made from materials with narrow hysteresis loops, such as silicon steel, rather than hard magnetic materials with wide loops

15.6

For magnetic recording, the field must be on the order of the coercivity of the recording medium. For a typical recording medium like $\gamma-Fe_2O_3$ with coercivity of approximately 300 Oe (23,873 A/m), we can calculate the required ~~amount~~ current.

The magnetic field at a distance r from a long, straight wire carrying current I is:

$$B = \frac{\mu_0 I}{2\pi r}$$

Rearranging to find the current:

$$I = \frac{B \cdot 2\pi r}{\mu_0} = \frac{H \cdot \mu_0 2\pi r}{\mu_0} = H \cdot 2\pi r$$

For a recording head at distance $r = 0.01m$ from the medium:

$$I = (23{,}873\ A/m) \times 2\pi \times (0.01m) \approx 1{,}500\ A$$

This current would be impractically large for a typical recording head, which is why actual recording heads use ferromagnetic cores to concentrate the magnetic field, reducing the required current by several orders of magnitude. Additionally the recording head is positioned much closer to the medium (typically micrometers rather than centimeters), further reducing the required current.